

NARSIS Workshop



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Methods for the derivation of fragility functions

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Introduction

Fragility function = Probability of reaching or exceeding a damage state given a level of loading

- □ Probabilistic tool (→ uncertainty treatment)
- □ To be coupled with probabilistic hazard assessment outcomes
- Essential component of Probabilistic Safety Assessment

Fragility functions may be:

- Empirical (from observed past events)
- **Experiment-based**
- ❑ Analysis-based

Most common in nuclear applications

- Current practical methods and assumptions?
- ➔ Which types of uncertainty to consider?



- 1. General principles
- 2. State-of-the-art of current methods
- 3. Selection of seismic intensity measures
- 4. Multi-variate fragility functions
- 5. Application
- 6. Beyond the lognormal assumption



Notation

$\label{eq:capitals} \Box \ \textbf{Capitals} \rightarrow \textbf{random variables}$

- IM = random Intensity Measure
- $im \rightarrow$ user-specified value (e.g. to define a threshold)

Accents

- Median $\rightarrow \overline{\blacksquare}$, e.g. \overline{im}
- Regression estimate $\rightarrow \widehat{\blacksquare}$, e.g. \widehat{EDP}



General Principles

or

Conditional probability

$$P_f(im) = P(DS \ge ds | IM = im)$$

$$P_f(im) = P(EDP \ge EDP_{th}|IM = im)$$

DS = Damage State

 $(DS \ge ds \Leftrightarrow EDP \ge EDP_{th})$

- **EDP = Engineering Demand Parameter**
- IM = Intensity Measure
- The lognormal assumption

$$P_f(im) = \Phi\left(\frac{\ln im - \ln \alpha}{\beta}\right)$$

 α = median (= \overline{im})





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General Principles

> Two major types of uncertainty

- Aleatory: inherent variability/randomness of physical quantities (e.g. variability of ground-motion for seismic analysis)
- Epistemic: lack of complete knowledge, incomplete data or modelling assumptions
 Reducible with the second secon

Reducible with additional measures or testing

Irreducible

Main causes of epistemic uncertainties

- □ In-situ uncertainty
- Modelling uncertainty
- Loading protocol uncertainty
- ☐ Finite sample uncertainty

Especially for experiment- or analysis-based fragility functions



General Principles

The « double-lognormal » model



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Fragility can be viewed as the result of a scaling exercise

- \succ Random variable $A\colon \Omega \mapsto \mathbb{R}$
- Sequence of EQ of increasing intensity:
 A: peak ground acceleration of the least intense EQ leading to failure of the SSC







Double log-normal model Random experiment performed in two steps

$$A = \left(\breve{A} \cdot \varepsilon_U \right) \cdot \varepsilon_R$$

A: Median value of the **population of scaling factors** that lead to onset of failure

$$\varepsilon_R \sim LN(0, \beta_R) \qquad \varepsilon_U \sim LN(0, \beta_U)$$



Example of probability density functions (PDF) $\beta_R = 0.2, \beta_U = 0.3$

Randomness and uncertainty modeled as two distinct (subsequent!) random experiments:

$$\varepsilon_R: \Omega_1 \mapsto \mathbb{R}$$

 $\varepsilon_U: \Omega_2 \mapsto \mathbb{R}$

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Separation of variables (Safety Factors)

Numerical analyses

Hybrid methods (Bayesian udpating)

Separation-of-Variables Method

$$\operatorname{Fr}(a) = \Phi\left(\frac{\ln\left(\frac{a}{\breve{A}}\right)}{\beta}\right)$$

a, $A \rightarrow$ ground motion (peak ground acceleration) $\beta \rightarrow$ variability (logarithmic standard deviation) $\Phi \rightarrow$ cumulative standard normal distribution function Scaling (of SSE loads):

 $\breve{F}_C = \breve{F}_S \cdot \breve{F}_\mu$

 \check{F}_S

Convenient because we can use

of the cases in NPP practice

of UHS is different from SSE!

Tolerated for rock sites

results from deterministic calculations (SSE \rightarrow safe shutdown earthquake \rightarrow , design earthquake"); \rightarrow reason why this method has been used in 99%

Caution for soil sites, if spectral shape

Capacity factor

- Strength factor
 - Failure mode dependent (ultimate stress, elastic limit, deformation, ...)

 (α)

 $\breve{A} = \breve{F}$

A_{SSE}

 $\breve{F} = \breve{F}_{RS} \cdot \breve{F}_{RE} \cdot \breve{F}_{C}$

- $S \rightarrow strength$
- $P \rightarrow$ demand (e.g. stress); $P_{T,SSE} \rightarrow$ total demand in case of SSE
- $P_N \rightarrow$ demand under normal conditions (no seismic loads)

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Separation-of-Variables Method

Why subtract non-seismic loads in strength factor?



 F_S always larger than F_D because – besides scaling σ_{SSE} - we are also scaling σ_N . (Red box will be full **EARLIER**.)



Separation-of-Variables Method EPRI (1994) « Divide et impera » EPRI (2003) EPRI (2009)

- Assumption of a given design level IM_s
- The safety factor F represents the design margin

 $\alpha = IM_s \cdot F$

Decomposition of *F* for a structure subjected to seismic loading:

 $F = F_{S} \cdot F_{\mu} \cdot F_{SR}$ Strength / Energy dissipation / Structural response

$$F_{SR} = F_{SA} \cdot F_{GMI} \cdot F_{\delta} \cdot F_{M} \cdot F_{MC} \cdot F_{EC} \cdot F_{SSI}$$

Decomposition of the dispersion term (quadratic combination):

$$\beta_{U} = \sqrt{\beta_{S}^{2} + \beta_{\mu}^{2} + \beta_{SA}^{2} + \beta_{GMI}^{2} + \beta_{\delta}^{2} + \beta_{M}^{2} + \beta_{MC}^{2} + \beta_{EC}^{2} + \beta_{SSI}^{2}}$$

Same formulation for β_R



Numerical Analyses

> Overview of the problem



Cornell et al. (2002)



 $\ln \widehat{EDP} = a + b \ln IM + \varepsilon$

Numerical Analyses

Least-Squares regression on the IM-EDP cloud



Identification of fragility parameters:

Linear relation between the logarithms

→ adequacy with the lognormal assumption

$$\begin{cases} \alpha = \exp\left(\frac{\ln EDP_{th} - a}{b}\right) \\ \beta = \frac{\sigma}{b} \end{cases}$$

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Shinozuka et al. (2000)



Numerical Analyses

Maximum Likelihood Estimation (MLE)

EDP → binary damage variable *Y*:

$$\begin{cases} y_i = 1 & \text{if } edp_i \geq EDP_{th} \\ y_i = 0 & \text{if } edp_i < EDP_{th} \end{cases}$$

Assumption: binomial or Bernoulli distribution for Y

\rightarrow Expression of the likelihood function of the fragility parameters α and β , given *N* data points:

$$L(\alpha,\beta) = \prod_{i=1}^{N} \left[P_f(im_i,\alpha,\beta) \right]^{y_i} \left[1 - P_f(im_i,\alpha,\beta) \right]^{1-y_i}$$

$$\{\hat{\alpha}, \hat{\beta}\} = \arg \max_{\alpha, \beta} L(\alpha, \beta)$$

Optimisation problem



Numerical Analyses

Generalised Linear Method (GLM) regression

EDP → binary damage variable Y:

$$\begin{cases} y_i = 1 & \text{if } edp_i \geq EDP_{th} \\ y_i = 0 & \text{if } edp_i < EDP_{th} \end{cases}$$

Fitting a linear combination of the input (based on Y):

Wang et al. (2018)



Bayesian Updating

Application of Bayes' rule



0.2

0.1

-5

0

A_m

0.4

0.2

0 └ -10

-5 A

0

5

-5

0

A

5

0.2



Summary of Methods

Method	Added value	Main limits	Example
Separation- of-Variables	 Reuse existing design calculations (high level of quality assurance!) -> cost-effective, good enough for vast majority of components; 	 Assumes linearity of demand w.rt. IM (partial correction with inelastic energy absorption factor); 	- EPRI TR-103959 (1994)
Regression "on a cloud"	 Simple and intuitive approach; Stable fragility estimates may be obtained with a few data points; 	 Constrained by the functional form of the IM-EDP relationship; Constant standard-deviation over the IM range; 	 Seismic fragility of an RC structure (Seyedi et al., 2010)
MLE / GLM regression	 Applicable to empirical fragility assessment (if only damage data are available); Ability to treat complete damage/collapse cases (where EDP values are usually inaccurate); Compatible with multivariate regression; 	 Loss of information (i.e., the true values of the EDP are not used); More data points are required to achieve stable fragility estimates; 	 Seismic fragility of a masonry structure (Gehl et al., 2013); Empirical tsunami fragility of buildings (De Risi et al., 2017);
Bayesian updating	 Compatible with expert-judgment approaches or Experimental results; 	 Influence of the prior distribution on the final fragility estimates; 	- Seismic fragility of switchgear cabinets (Wang et al., 2018)



Complexity of the ground-motion time histories



- Energy content?
- Duration of strong motion?
- Number of loading cycles?



Selection criteria

Efficiency: ability of an IM to induce a low dispersion in the distribution of the structural response

 $\ln \widehat{EDP} = a + b \ln IM + \varepsilon$

Low $\sigma_{\varepsilon} \rightarrow$ High efficiency

□ *Sufficiency*: ability of an IM to "carry" the characteristics of the earthquake that generated the ground motion

If
$$P(EDP|\ddot{x}_g) = P(EDP|IM(\ddot{x}_g)) \rightarrow IM$$

IM is sufficient

□ *Practicality*: strength of the link between IM and EDP

 $\ln \widehat{EDP} = a + b \ln IM + \varepsilon$

Large $b \rightarrow$ High practicality



Selection criteria

□ *Proficiency*: combination of practicality and efficiency

 $\ln \widehat{EDP} = a + b \ln IM + \varepsilon$

Large b/ σ_{ε} ratio \rightarrow High proficiency

□ Computability or Hazard compatibility: ability to compute the selected IM accurately with current GMPEs

Computability grade	IM
1	PGA, PGV, AI, SA(T), RSD75, RSD95
2	PGD, ASI, SI, NED, JMA, CAV, NCy
3	ARMS, A95, SL75, SL95, SMA, SMV, DCy

- 1. IM associated with many well-constrained GMPEs (good estimation of the epistemic uncertainty)
- 2. IM associated with few well-constrained GMPEs (difficulty to judge the epistemic uncertainty)
- 3. IM associated with no reliable GMPEs



Other methods to evaluate IMs / models Akaike Information Criteria (AIC):

 $AIC = 2k - 2\ln L$

k = number of parameters *L* = likelihood function Evaluates goodness-of-fit

□ ROC (Receiver Operating Characteristics) analysis:



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Motivations

- □ Case of <u>seismic loading</u>: no single IM can perfectly fulfil the conditions of *efficiency* and *sufficiency*
- Other <u>hazard loadings</u> (e.g., flooding, wind): a combination of IMs is usually required (e.g., velocity, height)





Multi-variate seismic fragility of a bridge system



Li et al. (2014)

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Vector-IM fragility functions (or "fragility surfaces")

Functional form:

:
$$P_f(im_1, im_2) = P(DS \ge ds | IM_1 = im_1, IM_2 = im_2)$$

= $\frac{1}{2} [1 + \operatorname{erf}(c_1 + c_2 \ln im_1 + c_3 \ln im_3)]$

\rightarrow Multi-variate GLM regression to estimate c_1 , c_2 , c_3

Possibility of defining a composite IM:

$$im_V = im_1^{\frac{c_2}{c_2+c_3}} \cdot im_2^{\frac{c_3}{c_2+c_3}}$$

 $P_f(im_1, im_2) = P_f(im_V) = \Phi\left(\frac{\ln im_V - \ln \alpha_V}{\beta_V}\right)$



$$\begin{cases} \alpha_V = \exp\left(-\frac{c_1}{c_2 + c_3}\right) \\ \beta_V = \frac{1}{(c_2 + c_3)\sqrt{2}} \end{cases}$$

By identification

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Vector-IM fragility functions (or "fragility surfaces")

- Steeper "slope" of the vector-IM fragility functions (reduction of the record-to-record variability)
- Need for a careful selection of IMs (issue of cross-correlation between IMs)



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Correlation map of various IM types (*Pellissetti et al.*, 2019)





Application

> Model of a PWR main steam line

- **Coupled model of a supporting structure and a steam line**
- □ CAST3M model from *Rahni et al.* (2017)
- Derivation of fragility functions accounting for epistemic uncertainties and record-to-record variability

containment building

steel steam line

Mode #	Period [s]
1	0.38
2	0.38
3	0.15
4	0.14
5	0.14





Rahni et al. (2017)



Application

Variation of structural and geometrical properties

Variable	Description	Uniform distribution interval
EIC	Young's Modulus - Inner containment	[27700-45556] MPa
ζ́rpc	Damping ratio - reinforced prestressed concrete	[4-6]%
<i>Č</i> RC	Damping ratio - reinforced concrete	[6-8]%
e1	Pipe thickness – Segment #1	[29.8 – 38.3] mm
e ₂	Pipe thickness – Segment #2	[33.3 – 42.8] mm
e3	Pipe thickness – Segment #3	[34.1 – 43.9] mm
e 4	Pipe thickness – Segment #4	[33.3 – 42.8] mm
e5	Pipe thickness – Segment #5	[53.4 – 68.6] mm
<i>e</i> 6	Pipe thickness – Segment #6	[34.1 – 43.9] mm
ζsl	Damping ratio – steam line	[1-4]%

Generation of 360 model samples



Application

Selection of ground-motion records

- □ Conditional spectrum approach (Lin et al., 2013)
- **Conditioning period** $T_1 = 0.38s$
- □ Selection for the PEER database (PEER, 2013)



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Application

Non-linear time-history analyses 360 data points



Failure criterion: load applied at the vertical stop = 400 kN



Application

> Selection of single IMs

IM	β	AIC	AUC
SA(0.14s)	0.5415	229.91	0.9166
SA(0.29s)	0.3898	175.82	0.9547
SA(0.50s)	0.5144	206.12	0.9363
PGA	0.4403	198.45	0.9399
PGV	0.4928	205.73	0.9381
PGD	1.2622	308.93	0.8469
AI	0.7674	182.13	0.9485
A95	0.4041	192.88	0.9389
SL75	0.9471	206.91	0.9325
SL95	0.7681	176.59	0.9508
SI	0.5293	213.94	0.9347
ASI	0.3775	176.23	0.9519
RSD75	-	-	-
RSD95	-	-	-
NCy	-	-	-
DCy	0.8123	191.58	0.9412
NED	1.4284	259.27	0.8968
CAV	0.6012	247.15	0.8951
ARMS	0.5728	240.45	0.9073

IM	Definition
PGA	Peak Ground Acceleration
PGV	Peak Ground Velocity
PGD	Peak Ground Displacement
AI	Arias Intensity
SA(T)	Spectral Acceleration at period T
A95	Level of acceleration that contains 95% of the Arias intensity
SL75 (95)	Slope of the Husid plot (cumulative AI over time) between 5% and 75% (and 95%) of the total AI
ARMS	Root-Mean-Square Acceleration: square-root of the integral of squared acceleration over time
ASI	Acceleration Spectral Intensity: integral of SA between two periods (here, 0:1s and 0:5s)
DO:	Cyclic Damage parameter: sum of the squared amplitude of all half-cycles, with the
DCy	rainflow counting method
NCy	Number of effective Cycles: the same as DCy, except that the half-cycles' amplitudes are normalized by the amplitude of the largest half-cycles in the signal
RSD75 (95)	Relative Significant Duration: length of time interval between when AI first exceeds 5% of total value and when AI first exceeds 75% (and 95%) of total value
SI	Housner Spectral Intensity
NED	Normalised Energy Density: integral over time of the squared ground velocity
JMA	Japanese Meteorological Agency instrumental intensity
SMA (SMV)	Sustained Maximum Acceleration (and Velocity): the third highest value of the absolute maximum acceleration (and velocity)
CAV	Cumulative Absolute Velocity: integral over time of the absolute acceleration time- history

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Application

Selection of vector IMs

IM ₁	IM ₂	βν	AIC	AUC
SA(0.14s)	SA(0.29s)	0.3724	173.71	0.9568
SA(0.14s)	SI	0.3464	167.19	0.9508
SA(0.29s)	SA(0.50s)	0.3834	173.74	0.9571
SA(0.29s)	PGA	0.3370	161.33	0.9424
SA(0.29s)	PGV	0.3718	171.22	0.9591
SA(0.29s)	AI	0.4687	171.69	0.9530
SA(0.29s)	SI	0.3659	167.43	0.9610
SA(0.29s)	RSD95	0.4371	174.11	0.9559
SA(0.50s)	PGA	0.3348	158.25	0.9439
PGA	PGV	0.3389	166.18	0.9532
PGA	AI	0.5027	170.79	0.9468
PGA	SI	0.3225	155.43	0.9339
PGA	ASI	0.3447	169.06	0.9416
PGV	ASI	0.3668	173.61	0.9526

Slightly better performance than single IMs

Some irrelevant parameters as single IMs now become useful (RSD)

→ Interesting combinations: [SA(0.29s) - PGA] [SA(0.50s) - PGA] [SA(0.29s) - SI] [PGA - SI]



Iso-probability lines of some fragility surfaces



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Uncertainty due to record-to-record variability





Uncertainty due to statistical estimation of parameters

Bootstrap sampling to estimate the variation of the median parameter



Very small contribution to the total uncertainty (enough data points?)

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 $\ln im_2$



Application

> Uncertainty due to mech. and geom. parameters

Multi-variate GLM regression, adding 3 thickness parameters (e₂, e₄, e₆)

Parameter	Regression coefficient	Std. Error	<i>p</i> -value (Wald statistic)	
c ₁ - Intercept	-22.008	4.030	4.73e-8	
$c_2 - SA(0.29s)$	1.991	0.359	2.98e-8	$g[r_f]$
c3 - PGA	1.284	0.342	1.75e-4	$= c_4 + c_2 \ln im_4 + c_3$
c_4 - Thickness e_2	95.504	46.303	0.0392	$-c_1+c_2 m m_1+c_3$
c₅ - Thickness e₄	135.553	46.184	0.0033	$+ c_4 e_2 + c_5 e_4 + c_6 e_6$
c6 - Thickness e6	98.819	44.217	0.0254	
40 35 30 -	0.95		0.8	





Decomposition of uncertainty terms



Link with HCLPF formulation:

 $SA(0.29s)_{HCLPF} = \alpha_{SA} \cdot \exp[-(\beta_R + \beta_U) \cdot \Phi^{-1}(0.95)]$

\rightarrow Reducing β_U leads to a larger HCLPF

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Distribution of the structural response of bridge columns



Karamlou & Bocchini (2015)



Validity of lognormal assumption?

> Seismic fragility functions of a steel frame structure



Mai et al. (2017)



Validity of lognormal assumption?

> Comparison of fragility models for the NPP steam line



Rohmer et al. (2019)

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Validity of lognormal assumption?

Distribution of structural capacities (from IDA)



Zentner et al. (2017)



Conclusions

- In NPP practice, fragility based on a single IM (typically PGA) is the main-stream (and completely sufficient for most buildings / components)
- Traditionally, separation-of-variables has been used in the NPP fragility community, for convenience (and because it is often sufficient for most SSC)
- For specific SSC (limited margin, non-linear response) alternative "numerical" methods have been developed
- A single IM is typically not sufficient to predict the value of the relevant demand parameter of a structure or component (e.g. base shear, bolt stress, ...)
- Vector-valued fragility take into account more than one IM and can help to reduce the variability
- In order to be beneficial for NPP practice, i.e. applicable to seismic design and seismic risk analysis, it is necessary that the results from PSHA are also presented / post-processed in vector-valued form

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Any questions?





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