Modelling External Hazards: Extreme Value Modelling

Hugo Winter

EDF Energy UK R&D Centre

2nd September 2019

Outline

Introduction and aims

Extreme value analysis

- Block maxima
- Threshold exceedances
- Estimation

Example application

Conclusion

Aims of this session

1. Provide an introduction to extreme value analysis.
2. Understand how these statistical approaches can be applied to data for external hazards.
3. To outline some common pitfalls when undertaking an extreme value analysis.
4. To provide you with a set of packages and functions that can be used to undertake and extreme value analysis.

Motivation: modelling external hazards

- Safety analysis for assessing the robustness of a nuclear power plant to external hazards is necessary for design and safe operation.

- We need to ensure we are protected for hazard levels that are more extreme than those observed in the data - especially given likely future climate changes.

- Need a method that is generalisable to several different external hazards to ensure consistency.

- For atmospheric and marine hazards a commonly used approach is **extreme value analysis**.

What is extreme value analysis?

- A statistical approach for analysing extreme data values of a variable of interest.
- First mentioned in 1928 by Fisher and Tippett.
- Formalised into statistical methods by Gumbel in paper in 1958.
- Use for environmental problems introduced in the 1950's.

Left: Roland Fisher; Centre: Leonard Tippett; Right: Emil Gumbel

Why use extreme value analysis?

- Provides a mathematically rigorous framework for modelling extreme values.
- Data are by definition sparse.
- Empirical approaches based upon the observed data an only provide accurate results within the range of the observed data $\rightarrow$ we often wish to extrapolate to higher levels.
- Different statistical models can lead to different tail behaviours $\rightarrow$ can often be too light-tailed and underestimate the probability of extreme events.
- Many statistical models are driven by average values as opposed to extreme values.

When to use EVA?

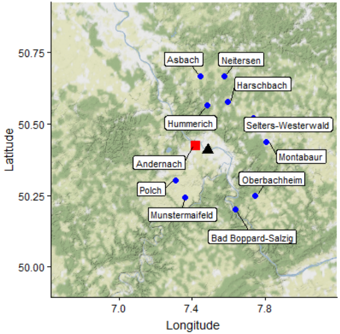
- When the variable of interest is stochastic (as opposed to deterministic) $\rightarrow$ e.g. storm surge ✓, tide ✗.
- When physical models are unavailable or unrealistic.
- When interested in obtaining estimates for extreme quantities that lie outside the range of the observed data.
- When there are at least 20-30 years of observations.

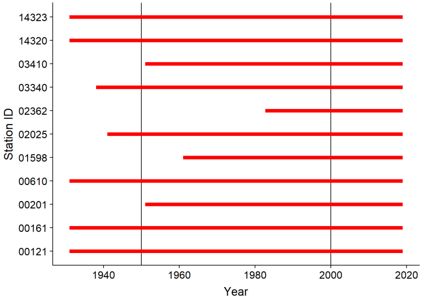
Modelling univariate extreme values

- Two main approaches exist for modelling univariate (one-dimensional) extreme values:
  - Block maxima
  - Threshold exceedances
- Block maxima methods were first to be developed.
- Threshold exceedance methods are most commonly used now.
- Modelling strategies for both assume observations are independent and identically distributed (IID).

My recurring rainfall data example

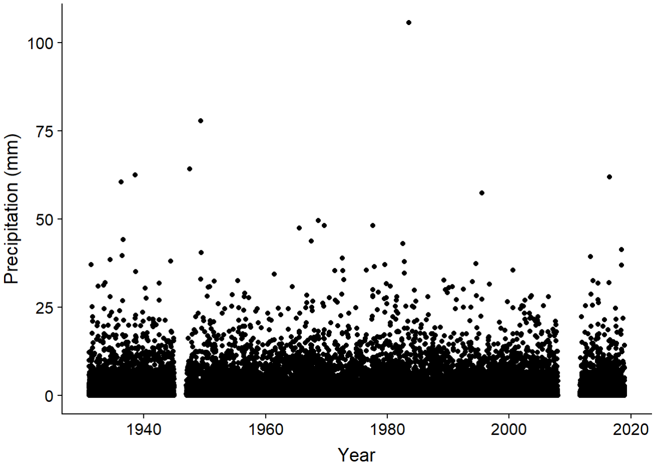
Daily rainfall accumulations (mm) in the vicinity of Mulheim-Karlich nuclear power plant.





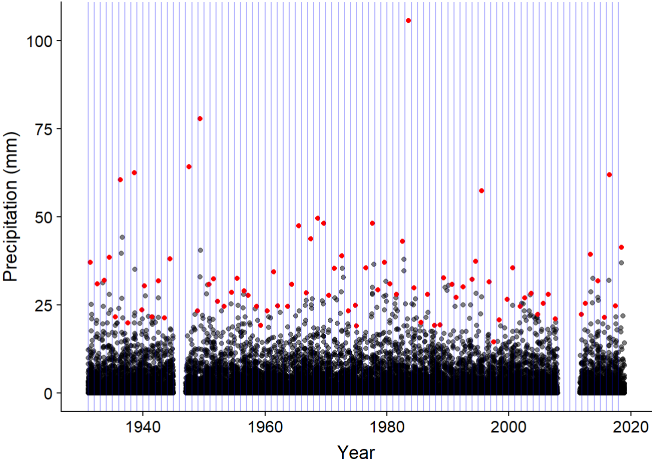
My recurring rainfall data example

We shall focus on data from the weather gauge at the Andernach weather gauge.



Block maxima

- Model the maxima of time periods of a certain length.
- Annual maxima often taken to remove the effect of seasonality.



Generalized extreme value distribution

Let $M_1, \ldots, M_n$ be random variables for the cluster maxima from n time blocks. The generalized extreme value (GEV) distribution can be used to model these maxima such that

$$G(x) = \mathrm{P}(M \leq x) = \exp\left\{-\left[1 + \xi\left(\frac{x-\mu}{\sigma}\right)\right]_{+}^{1/\xi}\right\},$$

for $1 + \xi(x - \mu/\sigma) > 0$ where

- $\mu \in (-\infty, \infty)$ is the **location** parameter
- $\sigma \in [0, \infty)$ is the **scale** parameter
- $\xi \in (-\infty, \infty)$ is the **shape** parameter

More detail on parameters

- The shape parameter ξ is a very important parameter in EVA.
- Controls the heaviness of the tail $\Rightarrow$ directly affects the extremes.
- The shape parameter of the GEV covers three different types of tail behaviour:
  - $\xi > 0$ - Fréchet distribution $\rightarrow$ Heavy upper tail
  - $\xi < 0$ - Negative Weibull distribution $\rightarrow$ Bounded upper tail
  - $\xi = 0$ - Gumbel distribution $\rightarrow$ Exponential upper tail

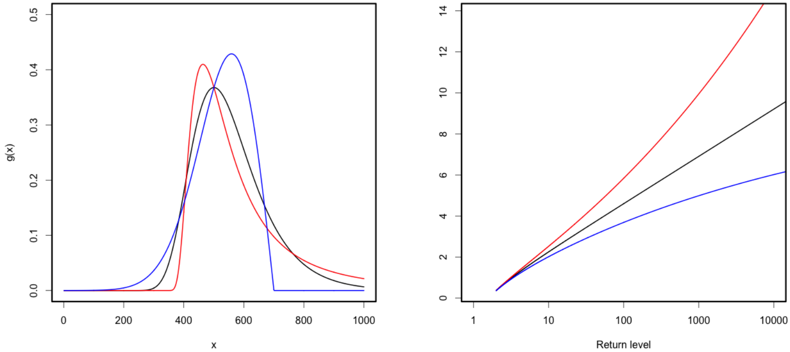
Return levels

- We are most interested in estimating the severity of extreme events.
- One way to summarise this is in terms of the T**-year return level** z_T.
- This is the event that happens once in every T years (i.e. has annual exceedance probability $1/T$).

For the GEV distribution fitted to annual maxima

$$z_T = \begin{cases} \mu - \frac{\sigma}{\xi}\left[1 - \{-\log(1 - 1/T)\}^{-\xi}\right] & \text{if} \quad \xi \neq 0 \\ \mu - \sigma \log\{-\log(1 - 1/T)\} & \text{if} \quad \xi = 0. \end{cases}$$

Effect of different shape parameters

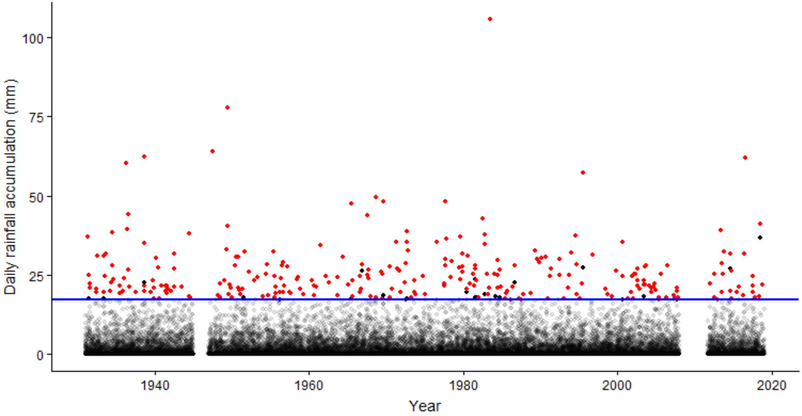


Left: GEV distribution function **Right:** Return level curves

Black: $\xi = 0$ **Red:** $\xi > 0$ **Blue:** $\xi < 0$

Threshold exceedances

- Model exceedances above a fixed high threshold.
- More efficient as more data are available but not necessarily independent.



Generalized Pareto distribution

Let $X_1, \dots, X_n$ be a sequence of random variables. The distribution G of the exceedances above a high threshold u is a generalized Pareto distribution (Davison & Smith 1990) of the form

$$G(x) = \mathrm{P}(X \le x \mid X > u) = 1 - \left(1 + \xi \frac{x-u}{\sigma_u}\right)_+^{-1/\xi},$$

for $x > u$ where

- $\sigma_u \in [0, \infty)$ is the **scale** parameter
- $\xi \in (-\infty, \infty)$ is the **shape** parameter

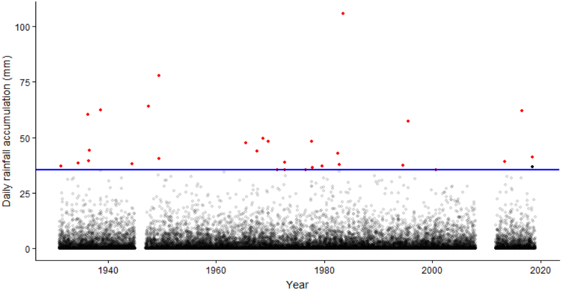
Choosing the threshold

- A bias variance trade-off exists when choosing the threshold.
- We wish to set the threshold low to use as many data points as possible in our analysis.
- Need the threshold set high enough for underlying limit assumptions of EV model to hold.

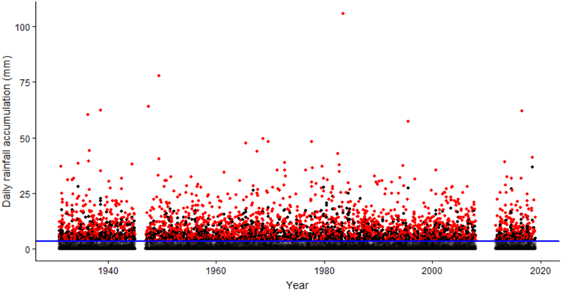
- **Threshold too high** $\Rightarrow$ not enough data, high uncertainty.
- **Threshold too low** $\Rightarrow$ non extreme data modelled, model not suitable.

Choosing the threshold

Threshold too high



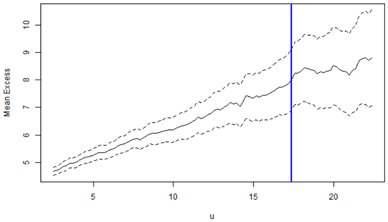
Threshold too low

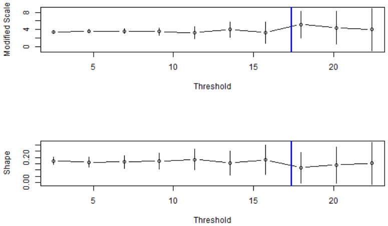


Choosing the threshold

- Two standard diagnostics exist for threshold choice:
  - Mean residual life (MRL) plot.
  - Parameter stability plots.

MRL plot **Parameter stability plots**





- Other approaches exists (e.g. Northrop et al. (2016))

Return levels

- Calculated in a similar way as for the GEV distribution.
- Since data are conditional upon having exceeded a high threshold we need to undo this conditioning by multiplying by $\lambda_u = \mathrm{P}(X > u)$.
- The *m*-observation return level is given below

$$z_m = \begin{cases} u + \sigma_u/\xi \left[(m\lambda_u)^\xi - 1\right] & \text{if} \quad \xi \neq 0 \\ u + \sigma_u \log\left(m\lambda_u\right) & \text{if} \quad \xi = 0, \end{cases}$$

where m must be sufficiently large to ensure that $x_m > u$. If n_T is defined as the number of observations in a year then $T = m/n_T$.

Declustering

- When fitting an extreme value distribution to data an important assumption made is that data are independent and identically distributed (IID).
- When using block maxima (for a sufficient block length) this is satisfied.
- This could be an issue for threshold exceedances tend to occur in clusters.
- If we model using all the exceedances it is likely that we will be overconfident and our confidence intervals will be too narrow.
- To solve this we usually undertake declustering to extract the peaks over the threshold (POTs).

Inference for univariate extreme value models

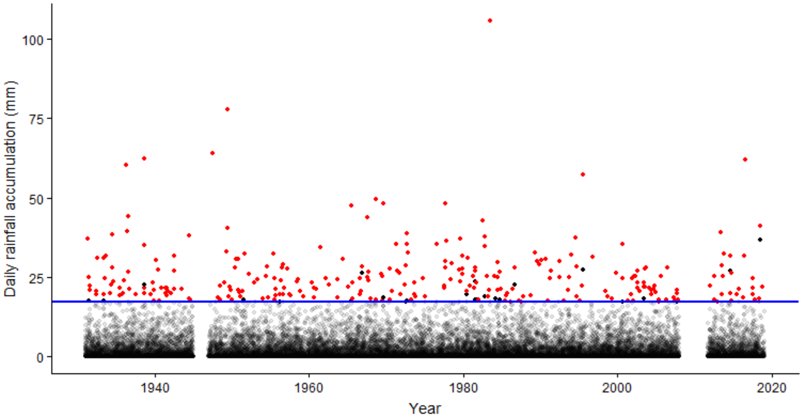
- Many different approaches exist for fitting both types of extreme value model:
  - Maximum likelihood $\rightarrow$ most commonly used.
  - L-moments $\rightarrow$ faster in certain situations.
  - Bayesian methods $\rightarrow$ modern approach, more computationally expensive.

- Many packages exist in `R` to fit extreme value models:
  - `evd` - Basic functions for an EVA
  - `extRemes` - Slightly more advanced set of functions
  - `ismev` - Companion package to Coles (2001)
  - `POT` - Peaks over threshold modelling
  - `evir` - More basic functions for an EVA

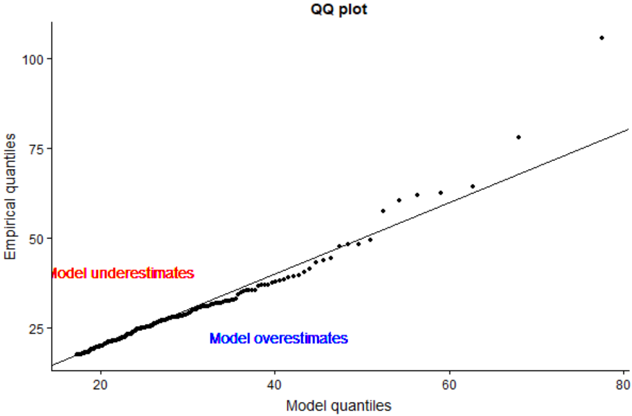
Confidence intervals

- Confidence intervals can be obtained in several ways:
  - Delta method
  - Profile likelihood
  - Bootstrapping (parametric and non-parametric)
- When looking at extreme quantities these intervals can get quite wide - this motivates methods for pooling data to obtain narrower intervals.
- Often profile likelihood or bootstrap intervals are preferred.

Data from Andernach site

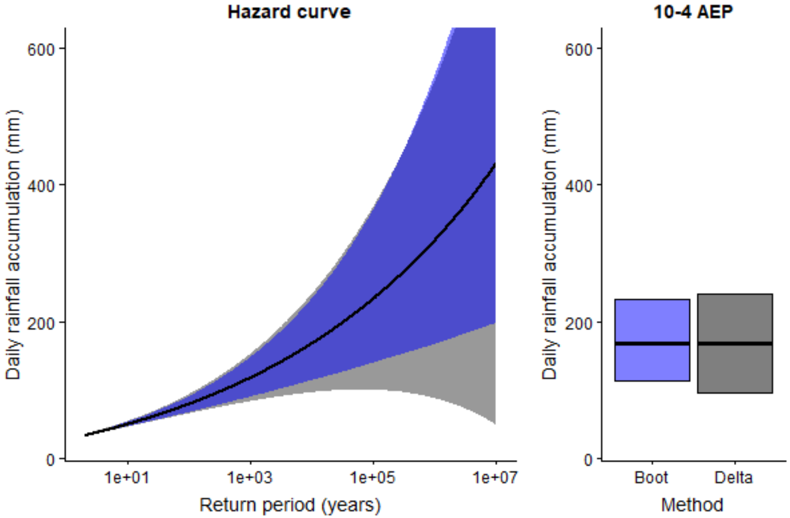


Model fit

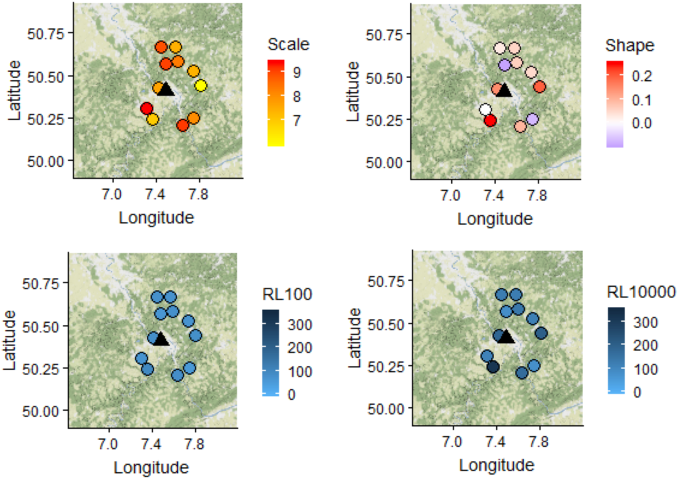


	Scale (σ_u)	Shape (ξ)
Best estimate	7.44	0.12
Standard error	0.63	0.06

Hazard curve



Multi-site analysis



Concluding remarks

- I hope that this course has given a basic overview of EVA.
- This area of statistics is seen as a vital skill across industry to estimate the risks posed by natural hazards.
- The go-to introductory book is still Coles (2001).
- Many packages are now out there (especially within R) to do this type of analysis.
- The application of such analysis is not straightforward due to modelling assumptions required - any questions then please ask me!

References I

Coles, S. G. (2001), *An Introduction to Statistical Modeling of Extreme Values*, Springer Verlag.

Davison, A. C. & Smith, R. L. (1990), 'Models for exceedances over high thresholds (with discussion)', *Journal of the Royal Statistical Society: Series B* **52**(3), 393–442.

Northrop, P. J., Attalides, N. & Jonathan, P. (2016), 'Cross-validatory extreme value threshold selection and uncertainty with application to ocean storm severity', *Journal of the Royal Statistical Society. Series C: Applied Statistics* pp. 93–120.